Fractal Dimension of a Coastline

Teacher’s Intro

OVERVIEW
Students will learn that the act of measuring the length of a natural shape such as a coastline depends on the scale of the ruler with which they measure it. This insight leads to the concepts of self-similarity and fractal dimension.

NM Math Standards:
5-7.M.1.2 Select and use appropriate units and tools to measure according to the degree of accuracy required in a particular problem-solving situation.
7.G.4.1 Compute the perimeter and area of common geometric shapes and use the results to find measures of less common objects.
8.G.4.4 Develop and use formulas for area, perimeter, circumference, and volume.
9-12.G.1.1 Understand that numerical values associated with measurements of physical quantities must be assigned units of measurement or dimensions; apply such units correctly in expressions, equations and problem solutions that involve measurements; and convert a measurement using one unit of measurement to another unit of measurement.
4.M.1.3 Identify the inverse relationship between the size of the units and the number of units.
5.M.2.3 Apply strategies and use tools for estimating and measuring the perimeter of regular and irregular shapes.
6.M.2.4 Select and justify the selection of measurement tools, units of measure, and degrees of accuracy appropriate to the given situation.
8.D.3.3 Conduct simple experiments and/or simulations, record results in charts, tables, or graphs, and use the results to draw conclusions and make predictions.
8.A.4.1 Use graphs, tables, and algebraic representations to make predictions and solve problems that involve change.

NM Science Standards:
Know that landforms are created and change through a combination of constructive and destructive forces, including: weathering of rock and soil, transportation, deposition of sediment, and tectonic activity similarities and differences between current and past processes on Earth’s surface (e.g., erosion, plate tectonics, changes in atmospheric composition) (6th)

Fractals are SMART: Science, Math & Art!

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Fractal Dimension of a Coastline

The field of fractal geometry began by asking the deceptively simple question: “How long is the Coastline of Britain?”

The answer is: it depends how closely you measure it! With a very small ruler, the length, or perimeter, is VERY large, as more and more details appear at small scales. Imagine measuring every rock, pebble and grain of sand on the beach!

Coastlines are usually not smooth, straight lines, but instead they have lots of similar detail at different sizes. Coastlines are fractals.

Why are coastlines fractals? Because they are formed by simple, repetitive processes, over thousands or millions of years. The crashing of waves slowly erodes the coastline. So does the rising and falling of the tides. Giant storms also erode the coastline, leaving behind fractal patterns.

On the following page, you will measure the length of the ‘coastline’ using three different lengthed rulers, and graph your results. First, cut out the paper ruler, which is approximately 10cm tall. You will use the long edge of the ruler first, then fold it in half to measure the perimeter at a 5cm scale, and then use the edge to measure the perimeter at a 2.5cm scale. In the chart, fill in your answers for each ruler scale, including first the number of units counted, and then the calculated perimeter. Next, graph your data on the axes. (You’ll have to choose an appropriate scale.)

What pattern or tendency do you observe that relates the scale of the ruler to the resulting perimeter measurement?
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**Fractal Dimension**

Measuring simple objects, Perimeter Whole number Dimensions

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**ASK:**

What is this object? An Amoeba? The coastline of Britain?

How long is the perimeter? (or any island, continent, amoeba, etc)

How can we quantify rough, jagged, bumpy natural objects?

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**REFLECT:**

What does this exercise tell us about the accuracy of precise measurements? What answer would we get with an extremely small ruler? "Perimeter" is a useful concept, but it has limitations.

The Fractal Dimension captures the essential nature of an irregular shape in a single number. The roughness of the shape is the same at any scale. Why do you think a coastline has this property? What natural process(es) might be responsible for the roughness being constant at different scales?

Is the dimension or 'jaggedness' really constant around the coastline?

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**CONNECT:**

The familiar concept of Perimeter is useful, but it breaks down for natural objects.

The familiar dimensions 1, 2, and 3 can be extended to describe infinitely complex natural objects.

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**DISCUSS:**

Plot the relationship between the perimeter and the size of your measuring device. The measured value increases with decreasing ruler size.

When plotted on a log scale, the (-) slope of this line is the Fractal Dimension. A more jagged edge has a higher Fractal Dimension, closer to 2, while a less jagged edge has a dimension closer to 1.

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**INVESTIGATE**

Using the image of Britain (or amoeba), measure the length of the coastline (or perimeter) using different lengths of a measuring device. Try measuring using 10cm, 5 cm, and 2cm rulers.

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**STANDARDS COVERED**

Measurement, Perimeter, Dimension, Accuracy [geography]

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**FUTURE INVESTIGATIONS**

Use software tools to measure perimeters using different scales.

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**For Tomorrow**

Learn more about fractional dimensions.
Quantify other coastlines and objects.
Explore fractal dimensions between 2 and 3.